An Approach to Mechanisms of Groundwater Flow and Rainfall Loss

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An Approach to Mechanisms of Groundwater Flow and Rainfall Loss

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Synopsis

A theoretical method for the estimation of the groundwater flow and the rainfall loss, which means the difference between the rate or the total quantity of rainfall and its runoff component, in the area of a small mountain-stream is here described. As the first step of the study, the method is discussed in relation to the following assumptions on which it is based. 1) There is a simple relation between the infiltration capacity, f, and the soil-moisture ratio, w, in the surface layer of a drainage basin which is expressed as follows; $f=f_c+(f_u-f_c)(w-w_h)/(w_s-w_h)$ for $w_h \leq w \leq w_s$, where w_s means the saturated soil-moisture ratio, w_h the hygroscopic coefficient, and f_u and f_c mean the upper and lower limits of infiltration capacity respectively. 2) The above equation derived for the infiltration capacity can be used for the recharge capacity, which is defined as the maximum recharge rate of groundwater and soil-moisture. 3) The recharge of groundwater occurs with the rate f_c only if the soil-moisture ratio in the surface layer equals or exceeds the maximum capillarywater capacity, w_c. 4) The discharge of groundwater flow is in proportion to the amount of water storage in the groundwater layer. 5) A simple relation exists between the evapotranspiration, E, and the soil-moisture ratio in the surface layer of the basin, w, as $E=E_p$ for $w \geq w_c$ and $E=E_p(w-w_h)/(w_c-w_h)$ for $w_c \geq w \geq w_h$, in which E_p means the potential evapotranspiration. 6) The depth of the imaginary surface layer related to the recharge capacity equals the actual depth related to evapotranspiration.

After various element considerations, the theoretical equations for the recharge capacity, the increment of groundwater flow, and the recovery of recharge capacity are derived. By applying the method to the Chama River with a catchment area of 1.08 km^2 at Kaikyo, the north part of Awaji Island, its usefulness has been made apparent.

1. Introduction

The analysis of the groundwater flow of any stream channel system gives us, needless to say, a very useful and encouraging clue to the problem of the utilization of water, and the problem of the loss capacity of rainfall defined as the difference between the rates of rainfall and of runoff as well.

Interesting papers on ground water flow have hitherto been published by several investigators. A notable approach to the physical mechanism of groundwater behavior in the simplified model of a basin was recently made by Ishihara and Takagi[1] On the other hand, a variety of research on the problem of rainfall excess in relation to flood runoff has been conducted by many hands since Horton's theory[2] of infiltration capacity was put into practice. Ishihara and his collaborators[3] show a very noteworthy and practical method, by means of which rainfall excess should be estimated by replaceing the concept of infiltration capacity with that of loss capacity, because a component of interflow must be included in the infiltration capacity. However, there has not yet appeared a study where the phenomena of both groundwater and the loss capacity of rainfall are analyzed consistently.

Now, the particular method which the writer puts forward here is based on the concept of recharge capacity which regarded as a part of infiltration capacity, and has dealt a twofold analysis of groundwater flow and rainfall excess in relation to direct runoff by plotting a simple stream channel system of the supposition of the primary relationship between the moisture content of the soil, recharge capacity and evaporation.

2. Recharge Capacity

When rain falls in a drainage basin after a considerably long dry period, part of the rainfall will infiltrate into the ground gradually, making the soil saturated. On the other hand, when the dry weather continues again from the time the rain ceases to fall, the water content of the soil will tend to be lost by evaporation. Such a process of infiltration-evaporation manifests does a very complex phase with a hysteresis, according to its initial condition and the boundary condition in each case[4].

Nevertheless, it is by no means an easy task at present to try to explain the runoff phenomena in a drainage basin of an extensive surface and there are a great many complex factors respecting such microformative phenomena which still contain too many unknown factors. Therefore, our first step will be to take the more macroformative approach from the standpoint of engineering.

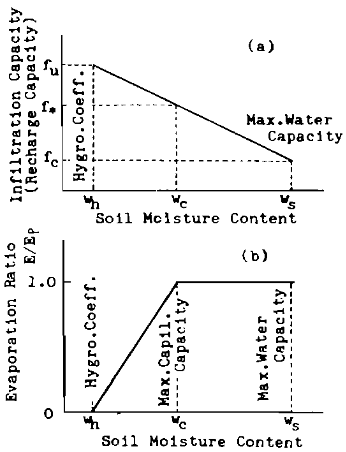

Fig. 1 Schematic diagrams of soil moisture content on (a) infiltration and or recharge capacity and (b) evaporation. E_p means potential evapotranspiration.

Now, the infiltration capacity of the rainwater into the soil f, which according to Neal's experiment[5] has a close relationship with the moisture content of the surface and the upper layers of the soil, can be expressed for practical purposes as in Fig. 1 (a). The infiltration capacity is governed by the soil moisture content of the upper layer, particularly that of the thin layer. Now, when the infiltration capacity of this layer happens to be f, providing that the rate of the increase of the soil moisture for the unit period of time caused by the rainfall with the intensity $i \geq f$ happens to be $f-f_c$, the following Horton equation[2] can be obtained from Fig. 1 (a).

$$f=f_c+(f_0-f_c)e^{-bt} \quad \cdots\cdots(1)$$

$$b=\frac{f_u-f_c}{W(w_s-w_h)} \quad \cdots\cdots(2)$$

in which f_u and f_c are the upper and lower limits of the infiltration capacity respectively; f_0 is the value of f for $t=0$, i. e. the initial infiltration capacity; w_s and w_h represent the saturated moisture ratio and the hygroscopic coefficient of the upper layer respectively; and W is the gross weight of the soil layer of the unit area relating to the infiltration capacity.

Now, most of the rainwater infiltrated into the soil moves in the form of lateral seepage or percolation, in the meantime forming the interflow; still, some of it contributes to groundwater storage in the form of vertical infiltration, providing the source of supply to the discharge of the groundwater flow; the remaining part is stored in the soil to increase the soil moisture content. But the rate of contribution to interflow in the infiltration capacity is very difficult to evaluate precisely whether the method of hydrograph analysis is used or the result of the infiltration test. Hence, for the present study such a maximum component as goes to increase the soil moisture content of the upper layer and to recharge groundwater storage will be defined as the recharge capacity f, excluding such a component as goes to become the interflow. This is represented by Eq. (1). In other words the rainfall excess $i-f(>0)$ corresponds to the direct runoff which contains the interflow.

3. The Groundwater Flow

The recharge intensity of groundwater storage in the recharge capacity can be considered to be f_c (constant), judging from the derived process of Eq. (1). However, it does not necessarily follow that the groundwater storage is increased as soon as the rain begins to fall. Although it is hard to make a precise estimate of the restriction of the movement of the water through the soil by gravitation, it will be taken for granted for practical purposes that the soil moisture content of the particular layer in question, w, should at least exceed the maximum capillarywater capacity w_c. In the same manner it must be considered that the groundwater storage will be increased even after the rain ceases to fall, if the condition $w \leq w_c$ exists.

On the other hand since the observational results of Dreibelbis's[6] finding that the soil moisture content of layers deeper than 10 cm from the surface appears to show no great change, it can be assumed that a soil moisture content close to the maximum capillarywater capacity is constantly being maintained here. Putting it in other way, it means that the groundwater is being supplied or increased almost at the same time when the surface soil comes to exceed w_c. Therefore, the recharge intensity of groundwater can be expressed as follows by taking the time when the rain starts to fall as its commencing point of time:

$$\left.\begin{aligned} f_g &= 0\,; \quad t<t_0, \quad w<w_c, \quad f>f_* \\ &= f_c\,; \quad t_0 \leq t \leq t_f, \quad w \geq w_c, \quad f \leq f_* \end{aligned}\right\} \qquad (3)$$

in which t_0 represents the time when the soil moisture ratio of the surface layer w becomes $w=w_c$ after the rain starts to fall; t_f represents the time when it becomes $w=w_c$ after the rain ceases to fall: and f_* is the value of f in the state of $w=w_c$.

The discharge of the groundwater flow from the drainage basin can be expressed as follows by plotting a simple model as shown in Fig. 3:

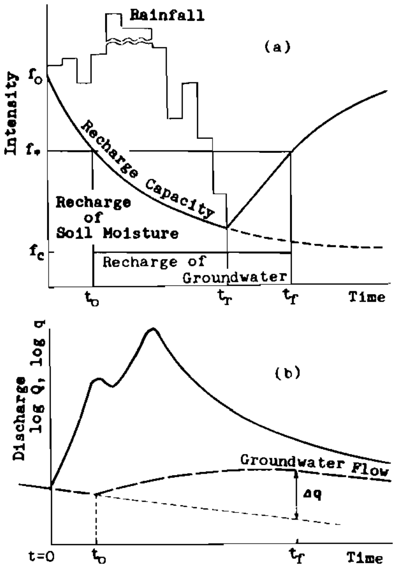

Fig. 2 (a) Distribution of recharge capacity
(b) Increment of groundwater flow

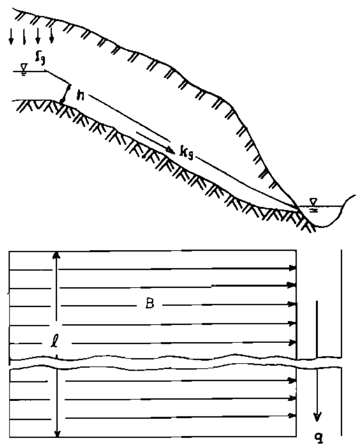

Fig. 3 Model of groundwater flow

$$q = k_g h l I \equiv MS \quad \cdots\cdots (4)$$

in which k_g is the coefficient of permeability of the layer in which the groundwater moves, h is its depth, l is the length of the stream channel and I is the potential gradient of the flow. Providing that I remains almost constant, q tends to be in proportion to the groundwater storage S; M is its proportional constant and corresponds to the recession constant of the groundwater flow.

On the one hand we can form the following equation for continuity, if f_g is converted into mm/hr, q into m^3/sec and the area of the drainage basin into km^2:

$$\left.\begin{aligned} \frac{dS}{dt} &= f_G - q \\ f_G &= \frac{Af_g}{3.6} \end{aligned}\right\} \quad \cdots\cdots (5)$$

From this, a fundamental equation for the groundwater flow is obtained as follows:

$$\frac{1}{M}\frac{dq}{dt} + q = f_G \quad \cdots\cdots (6)$$

Putting $q = q_0$ at $t = 0$,

$$q = f_G[1 - e^{-M(t-t_0)}] + q_0 e^{-Mt} \quad \cdots\cdots (7)$$

Since the second term in the right side of the above equation represents the discharge of the groundwater flow at the time when $f_g = 0$, the maximum quantity of the increase of the groundwater flow caused by the rainfall, i. e. Δq, can be expressed as follows:

$$\Delta q = f_G[1-e^{-M(t_f-t_0)}] \quad \cdots\cdots(8)$$

4. The Recovery of Recharge Capacity

When the rainfall ceases to fall, the soil moisture content will tend to decrease through the processes of downward infiltration and evapotranspiration during the period when the moisture ratio of the soil remains in the state of $w \geq w_c$ and through the process of evapotranspiration only, during the period after $w < w_c$, and the recharge capacity gradually will tend to be recovered. In regard to the relationship between the evapotranspiration and the moisture content of the soil, if the microformatiove approach is not taken too seriously, it appears for practical purposes that the relationship indicated[7] by Fig. 1 (b) holds true.

Generally speaking, whenever many experimental data are completed in this way, the problems we should unlock are how to make an adequate evaluation of E_p and how to decide the depth of the surface layer relating to the evapotranspiration so that the soil moisture ratio may be obtained. As a matter of fact, a similar problem to the latter existed in the case of infiltration capacity, too. But the depth of the surface layer relating to recharge capacity is not real but imaginary, even if its equivalent value may be obtained as a result of calculation, because the recharge capacity is regarded as the only part of the infiltration capacity which has physical meaning.

Therefore, for this purpose, the depth in both cases is assumed to be the same, from the standpoint of the water balance of the particula-layer in question. Then, on the basis of the geometrical relationships in (a) and (b) of Fig. 1, making the recharge capacity f_e at time $t=0$ when the rain comes to a stop and the recharge capacity f_* at time $T \equiv t-t_c=0$ i. e. $w=w_c$, the following equation can be formed as a formula showing the recovery of recharge capacity:

$$\left.\begin{aligned} w \geq w_c;\quad & f_0 = \frac{f_u - f_c}{W(w_s - w_h)}(E_p + f_c)t + f_e \\ w < w_c;\quad & f_0 = f_u - (f_u - f_*)e^{-CT} \end{aligned}\right\} \quad \cdots\cdots(9)$$

$$c = \frac{E_p}{W(w_c - w_h)}$$

in which W is the gross weight of the soil layer of the unit area relating to the evapotranspiration or the recharge capacity. Yet strictly speaking it is essential to make some alterations to the expression of the above equation, according to whether the rain comes to a stop during the night or the day. Nevertheless, the considerations given have refer to the average intensities during one day.

5. Analysis of Groundwater Flow and Rainfall Loss

In order to apply the forementioned theories to the practical analysis of groundwater flow and rainfall loss, the parameters in Eqs. (1), (8) and (9) must be estimated. Most of these are unobtainable unless painstaking calculations are made from the hydrograph.

(*i*) *The Recharge Capacity Curve*

The results of the observed hydrograph are drawn on semi-logarithmic paper and M is estimated from the recession part which seems to be the groundwater flow in the hydrograph. Assuming $t_0=0$ and $t_f=t_r$ as the first approximation, the total quantity of the direct runoff, Q, is sought, and then $F'=R-Q$ is calculated. Providing that this particular rain continues to fall with the intensity of $i\geqq f$ for its duration, the following equation can be established.

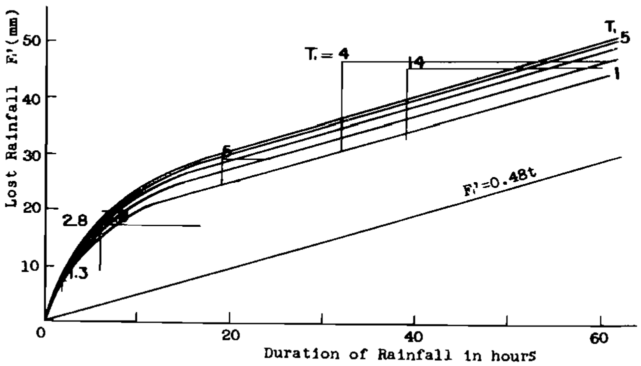

Fig. 4 $F'\sim t_r$ curves

$$F'\doteqdot F=\int_0^{t_r} f dt=f_c t_r+\frac{f_0-f_c}{b}(1-e^{-bt_r}) \quad \cdots\cdots(10)$$

Therefore, if the values of F' obtained from the number of storms are plotted for each t_r and are classified according to the length of antecedent dry period T', the $F'\sim t_r$ curve is obtained for each T' The parameters in Eq. (10) may be estimated from a curve for each T' That is, the first approximation of each constant in Eq. (1) can be determined. Furthermore, if the value of the upper limit of the recharge capacity f_u is utilized, which can be obtained as a result of the compiled curve for the recovery of recharge capacity to be described later, the first approximation of the standard recharge capacity curve which is shown by the following equation can be obtained :

$$f=f_c+(f_u-f_c)e^{-bt} \quad \cdots\cdots(11)$$

However, it is very difficult to assume a proper curve for $F'\sim t_r$ in the first stage, because not all actual rainfall always has an intensity of $i\geq f$, and in many case the time of the rainfall-stoppage is involved. Although it is hard at the present stage to make an adjustment for the assumption of $i\geq f$, the actual curve in the case of the duration time of rain t_r involving the time of stoppage t_r' can be drawn by considering the following three points plotted as follows: F' for t_r, F' and $F'-E$ for t_r-t_r', in which E is the quantity of evaporation during the period of the stoppage of the rainfall and this can be obtained from the available meteorological data.

(ii) The Recharge Capacity Recovery Curve

The first approximation for the recharge capacity recovery curve is obtained by plotting f_0 which can be estimated in the way described above on the basis of T' for that part of $T>0$. However, when the observed values of the soil moisture ratio w_s, w_c, w_h, etc. are available, an adequate adjustment should be made so that no contradiction of those values will occur. Another point calling for due attention in this connection is that the relation of $T' \doteqdot T$ can not be accepted whenever T' is found to be smaller, because in the strict sense T' should be $T+t_c$. The $T<0$, i. e. that part of $f<f_*$, is obtainable by calculation. Thus it is in this way that the standard recharge capacity recovery curve is to be made.

(iii) Groundwater Flow

The assumption of f_0 is essential in the initial stage of the pursuit of the discharge of groundwater flow, but errors that might be committed due to the assumed f_0 could be reduced to minimum if the pursuit could be made with rain which has fallen after a long antecedent dry period. Because the length of time required to attain the state of $f=f_*$ for each rainfall t_0, t_f can easily be estimated by making use of the standard recharge capacity curve and the standard recharge capacity recovery curve, Δq can be obtained from Eq. (8). The recession part of the groundwater flow can be estimated without any difficulty since M is known.

The value f at each moment for the rainfall of $i>f$ is obtainable directly from the standard recharge capacity curve, but that in the case of the rainfall of $i<f$ for the length of time $t_1 \sim t_2$ should be obtained from the following equations:

$$\left.\begin{array}{ll} f<f_* ; & \int_{t_1}^{t_2} i\,dt = \int_{t_1}^{t_2'} f\,dt \\ f \leq f_* ; & \int_{t_1}^{t_2} i\,dt = \int_{t_1}^{t_2} f\,dt + f_c(t_2 - t_2') \end{array}\right\} \quad \cdots\cdots(12)$$

that is, $f(t_2')$ could be regarded as the corresponding recharge capacity for time t_2.

All that is required to check the fitness of the discharge of groundwater flow which is pursued in the way described above by comparing with the forementioned observed hydrograph, in order to make adequate correction or adjustment of each value of the parameters in Eqs. (1), (8) and (9) step by step. Once the standard curves of both the recharge capacity and the recharge capacity recovery are ultimately obtained in this way, then the direct runoff component at each moment of rainfall can be obtained as $i-f(>0)$.

6. Applied Example

The method proposed here has not been put into practice by many rivers data yet, but the case of the Chama River ($A=1.08\ \mathrm{km}^2$) located in the northern part of Awaji Island can be taken as one example to illustrate how the method has been applied.This area is an eroded region mainly composed of granite, its surface layer having a depth of $10 \sim 30$ cm and being covered with copse. This region generally has an annual precipitation of 1,100 mm and small ponds lie scattered in great number in the drainage basin. Of course it is not a good example because

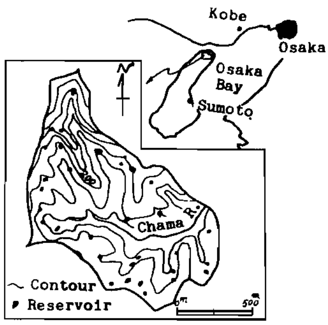

Fig. 5 General view of Chama River Basin

the particular data for the discharge in the stream can not be free from human factors on account the partial loss of discharge control through irrigation. But the discharge has been measured by a trapezoidal weir and its accuracy has been fairly satisfactory.

By drawing the results of the observed hydrograph on semi-logarithmic paper the following value of M has been arrived at from the characteristics of the recession part which can be considered to be composed from the groundwater flow only in the hydrograph:

$$M=0.116/day\,;\quad q\geq 0.006\, m^3/sec/km^2$$
$$=0.031/day\,;\quad q\leq 0.006\, m^3/sec/km^2$$

The reason why the value of M was found to be variable is not yet known. Although it appeared to be rather easier to explain this difference in terms of seasonal classification with due allowance for the effect of evaporation, in fact the classification by discharge was adapted, because the permeability of the layer of groundwater movement in this particular area would change to a great extent. On the basis of experimental results the following values were adopted: $w_s=34\%$, $w_c=20\%$ and $w_h=10\%$. And as the value of E_p, the value of pan-evaporation was adopted by classifying according to season, duly weighing the accuracy of computation.

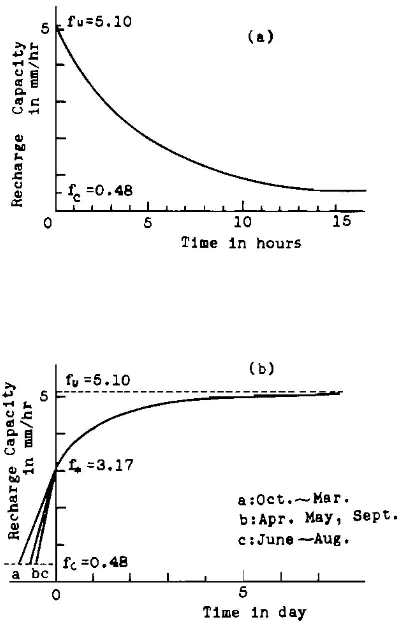

Fig. 6 Standard curves of (a) recharge capacity and (b) its recovery

The curves $F'\sim t_r$ which were used for the first approximation of the recharge capacity are shown in Fig. 4. Almost all of the data utilized for that purpose were those taken during June~August. The standard curves of the recharge capacity and its recovery which were ultimately estimated after repeated computations are shown in (a) and (b) of Fig. 6. In Fig. 6 (b) that part of $f\leq f_*$ was classified by seasons, but that part of $f\geq f_*$ was not. It is true that this part should be classified as well, but because no great difference in fact was observed, the problem of the evaluation of evapotranspiration has been left untouched to be tackled in the future.

The evaluation of the equivalent recharge capacity at the end of the time $t_1\sim t_2$ of $i<f$ was made by modifying Eq. (12) as follows:

$$\left.\begin{aligned} f>f_*\ ; \quad & \Delta f=\frac{f_u-f_c}{W(w_s-w_h)}\Delta R \\ f\leq f_*\ ; \quad & \Delta f=\frac{f_u-f_c}{W(w_s-w_h)}[\Delta R-f_c(t_2-t_1)] \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(13)$$

in which ΔR represents the quantity of the rainfall during the period of $t_1 \sim t_2$ and Δf the decreased quantity of f during that period.

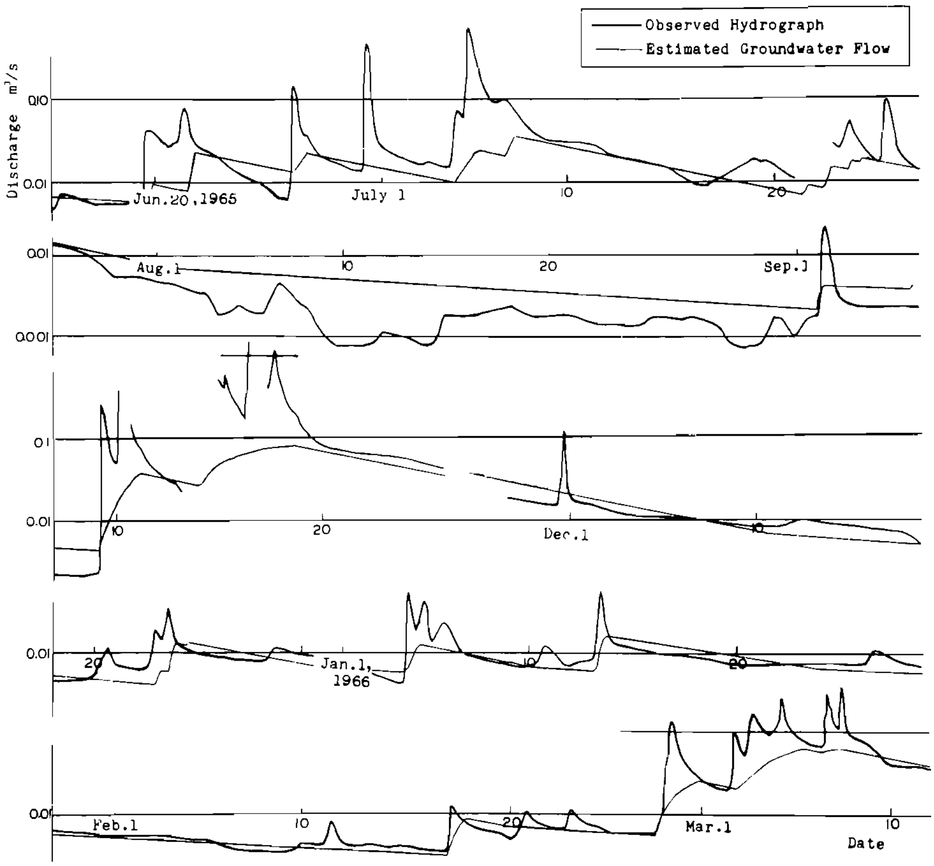

Fig. 7 Hydrograph for Chama River at Kaikyo, Awaji Island

Now, the thin line in Fig. 7 shows the result of the pursuit of the discharge of the groundwater flow by making use of Fig. 6 and Eq. 8 for the period starting from the later part of June 1965 and ending by the early part of March 1966: Thick lines in the same diagram is the observed hydrograph.

Next, on the supposition that the estimated value of the discharge of the groundwater flow obtained in the manner described above is correct, Fig. 8 was obtained by separating the direct runoff component Q from the observed hydrograph by obtaining the quantity of the lost rainfall from each rainfall $F'=R-Q$, and by

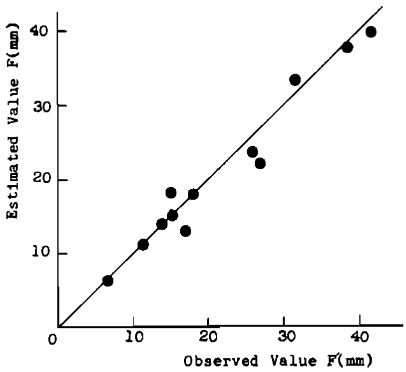

Fig. 8 Comparison between estimated and observed values of lost rainfall. (June, 1965~March, 1966)

comparing this result with the estimated value F derived from the forementioned theory. While doing so, whenever it was found that the water of the pools lying scattered in the basin gradually became empty because of the use of water for irrigation in the dry period and that the catchment area corresponding to the observed hydrograph mentioned above shrank to a great extent, the effective area of the basin A_a was adopted instead of the total area of the basin A_t for the above calculation and the relation $A_a/A_t=q_a/q_t=Q_a/Q_t$ was used, in which q means the discharge of the groundwater flow at each moment and Q the summarized value of the direct runoff with each moment and suffix a and t correspond to the adjusted and non-adjusted area of the basin respectively. The estimated groundwater flow shown in Fig. 7 had been drawn in consideration of the above fact.

Figure 8 shows the results pertaining only to $R<100$ mm because of the partial lack of observed data for the runoff at the time of heavy rainfall. But as far as Figs. 7 and 8 are concerned, they can give convincing proofs of the usefulness of the method proposed here.

7. Conclusion

This paper has proposed one method for conducting a consistent analysis of the groundwater flow and rainfall excess which contain the component of interflow, with a small mountain-stream as the object of study. But the recent findings of various research in the field of soil-physics have not always been duly appreciated in the formulation of fundamental concepts and they have only been put into practice in moderation at best. This point the writer is planning to discuss at some other time together with the resuls of a few fundamental experiments which are now being conducted. It is admitted that it was a simple stream channel system that was plotted for the present method and that the required computation is very laborious because the auto-recording data of rainfall are prerequisite, and these necessitate repeated trial calculations. So it is contemplated hereafter to find a way of simplifying the process of calculation and also of making an analysis of some large scale stream channel.

Acknowledgement

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